

A.2.7 Binomials Containing Radicals

Review:

Distribute and simplify.

$$(2a + 1)(3a - 4)$$

$$\begin{array}{r} 6a^2 - 8a + 3a - 4 \\ \hline 6a^2 - 5a - 4 \end{array}$$

$$(4x + 5)^2$$

$$\begin{array}{r} (4x+5)(4x+5) \\ 16x^2 + 20x + 20x + 25 \\ \hline 16x^2 + 40x + 25 \end{array}$$

$$(2x + 3)(2x - 3)$$

$$\begin{array}{r} 4x^2 - 6x + 6x - 9 \\ \hline 4x^2 - 9 \end{array}$$

Use the distributive property and then simplify the following expressions.

$$(4 + \sqrt{7})(3 + 2\sqrt{7})$$

$$\begin{array}{r} 12 + [8\sqrt{7} + 3\sqrt{7}] + \sqrt{49} \\ 12 + 11\sqrt{7} + 7 \\ 19 + 11\sqrt{7} \end{array}$$

$$(2\sqrt{3} - \sqrt{6})^2$$

$$\begin{array}{r} (2\sqrt{3} - \sqrt{6})(2\sqrt{3} - \sqrt{6}) \\ 4\sqrt{9} - 2\sqrt{18} - 2\sqrt{18} - \sqrt{36} \\ 4 \cdot 3 - 4\sqrt{18} - 6 \\ 12 - 4\sqrt{9}\sqrt{2} - 6 \\ 6 - 4 \cdot 3\sqrt{2} = 6 - 12\sqrt{2} \\ (a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d}) \end{array}$$

$$\begin{array}{r} a^2\sqrt{b^2} - ac\sqrt{bd} + ac\sqrt{bd} - c^2\sqrt{d^2} \\ a^2|b| - c^2|d| \end{array}$$

Binomials like $(a + b)(a - b)$ are called conjugates.The product of $(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d})$ will always be an integer if a, b, c , and d are integers.

Simplify the following expressions.

Multiply the numerator and denominator by the conjugate of the denominator to rationalize the denominator.

$$\frac{2}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$(1+\sqrt{3})(1-\sqrt{3})$$

$$1-\sqrt{3} + \sqrt{3}-3$$

$$1-3 = -2$$

$$\frac{2-2\sqrt{3}}{-2} = -1 \left(\cancel{2-2\sqrt{3}} \right)$$

$$= \boxed{-2+2\sqrt{3}}$$

$$\frac{4+\sqrt{5}}{\sqrt{2}-7} \cdot \frac{\sqrt{2}+7}{\sqrt{2}+7} =$$

$$\frac{4\sqrt{2} + 28 + \sqrt{10} + 7\sqrt{5}}{2-49}$$

$$= \frac{4\sqrt{2} + 28 + \sqrt{10} + 7\sqrt{5}}{-47}$$

If $f(x) = \frac{x+1}{x}$, find $f(\sqrt{3} + 2)$

$$f(\sqrt{3}+2) = \frac{(\sqrt{3}+2)+1}{\sqrt{3}+2}$$

$$= \frac{\sqrt{3}+3}{\sqrt{3}+2} \cdot \frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$= \frac{9-2\sqrt{3}+3\sqrt{3}-6}{3-4}$$

$$= \frac{3+\sqrt{3}}{-1}$$

$$= \boxed{-3-\sqrt{3}}$$